Explorations in Negative Voting

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Abstract

This voting system aims to emphasize, rather than electing the most popular candidate as most voting systems do, but instead not electing widely unpopular candidates. It does this by allowing voters to vote directly *against* candidates, rather than only *for* candidates. This paper shall explain the means by which the system operates, various traits it does and does not have, and how its outcomes in certain situations compare with those of other systems.

1 How it Works

This voting system operates similarly to *Cumulative Voting*. Each voter has a certain number of positive votes and a certain number of negative votes, and they spend them among the candidates. Positive votes are spent in the normal manner, negative votes in the opposite. The voter spends more negative votes on candidates they *dis*like more. This lowers the candidate's total score and reduces their chances of winning.

Alternatively, this system can be used as a semiproportional representation system. Simply give each of the n candidates c_i with a positive total score t_i representation equal to

$$\frac{t_i}{\sum_i^n t_i}$$

Of course, this requires enumerating the special case where $t_i = 0 \forall i$ of giving every candidate equal representation.

2 Traits it Has

Definition 1 *A voting system is said to be* consistent *if when divided into subsets and there is a certain candidate is the winner of every subset it appears in, then that candidate must win the election.*

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Since each candidate is assigned an integer value for their score, comparing any subsets is simply comparing integer values. If there is one score that is the maximum of every possible subset containing it, then it is the supremum of the entire set, and thus the winner of the election.

Definition 2 *A voting system is said to be* monotonic *if increasing the rank of a candidate always either helps that candidate or is neutral towards them, and decreasing their rank always hurts them or is neutral towards them.*

Increasing the rank of a candidate means giving them more positive votes or fewer negative votes, which will always increase their final score. This will either bump them up in the rankings, past some other candidate, or will be insufficient to move them past another candidate, but as it can never lower their score it can never lower their final ranking. The reverse is also true; lowering a candidate's score cannot help them.

Definition 3 *A voting system is said to satisfy* participation *if it is always advantageous to vote honestly, as opposed to not voting at all.*

Note that this says nothing about voting honestly as compared to voting dishonestly. Because the system is *monotonic* and there is no way to stop an election by not voting, it follows immediately that the system satisfies participation.

Definition 4 *A voting system is said to be* reversal symmetric *if every voter voting the opposite of the way they did in a previous election causes the results to be the opposite of what they originally were.*

Voting the opposite of how they did last time means replacing all the positive votes with negative ones and vice versa, and therefore simply multiplies the final scores by negative one, and therefore reverses the order of them.

3 Traits it Doesn't Have

Definition 5 *If there is a candidate who is the first choice by the majority of voters, then that candidate must be selected as the winner if the voting system is to satisfy the* majority *criterion.*

This is the whole point of the system. There are quite often candidates who are beloved by the slim majority but hated by the rest of the populace, and therefore electing them wouldn't serve to represent the wishes of the whole group. However, in this system, the minority voting against the majority's favorite candidate could allow a different, less popular but less hated candidate to win. We'll see an example of this later, in the *comparisons to other systems* section.

Definition 6 *If adding a candidate who is identical to the winner of an election cannot cause some third candidate to win, then the voting system is said to be* clone independent

Sadly, because of vote splitting, this does not hold for this system.

Definition 7 *If there is a candidate who wins every pairwise election, he must win the overall election if the voting system is to satisfy the* Condorcet Criterion

The fact that this system does not satisfy the *Condorcet Criterion* is demonstrated in the following section.

4 Comparisons with other systems

Suppose there is an election where three candidates are running. Two, the *Blue* and *Red* candidates, are widely known and total opposites, and their constituents loathe each other. The third, *Green*, is less widely known, less popular, but his constituents dislike the *Red* candidate thoroughly. Suppose 51% of the populace votes for *Red* and against *Blue*, 44% votes for *Blue* and against *Red*, and 5% vote for *Green* and against *Red*. Then the total votes for each candidate are $Red : 2\%$, $Blue : -7\%$, and $Green : 5\%$ and *Green* is the winner. In ordinal terms, this is $51\% Red > Green > Blue$, $44\% Blue > Green > Red$, $5\% Green > Blue > Red$. By any system that obeys the $Majority$ criterion, the outcome would have been *Red*. Similarly, if *Blue* had not run, *Red* clearly would have won, and so this system does not satisfy the *Condorcet Criterion*.

This system is clearly different from any ordinal system. Consider a case where two voters vote on four candidates, A, B, C, and D. The first voter votes $A : 4, B : 3, C : 1, D : -4$ and the second votes $A : 3, B : 1, C : -1, D : -3$. Any ordinal system would not be able to tell these apart, interpreting them both as $A > B > C > D$.

Now for the final scenario, where we will see the differences between this system and the standard *Cumulative Voting* system. Consider an election with 4 candidates, as above. Half of the populace loves A, likes B, is indifferent to C, and dislikes D while the other half likes D, is indifferent to C, and dislikes A and B. That is, 50% have the preference order $A \geq B \geq C \geq D$ and the other 50% have $D \geq C \geq B \geq A$. Under cumulative voting, where each voter is given 8 votes, these opinions are expressed as, for the first group: $A : 6$, $B : 2$, $C : 0, D : 0$. The second group votes $A : 0$, $B : 0$, $C : 0$, $D : 8$. These lead to the total scores $A : 6$, $B : 2$, $C : 0$, $D : 8$ and D is the winner.

Under the system presented here, assuming 4 positive votes and 4 negative votes to match the 8 original votes, the first group could vote $A : 3$, $B : 1$, $C : 0$, $D : -4$. The second group could vote $A : -2$, $B : -2$, $C : 0$, $D : 4$. These lead to the scores $A : 1$, $B : -1$, $C : 0$, $D : 0$ with A being the winner.

Using this system we have avoided the election of the widely unpopular D and instead elected the less charismatic but less divisive A, an outcome that may make fewer voters ecstatic but also will surely infuriate fewer voters. It is an outcome that is more representative of the wishes of the people as a whole, rather than as splintered factions.

References

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